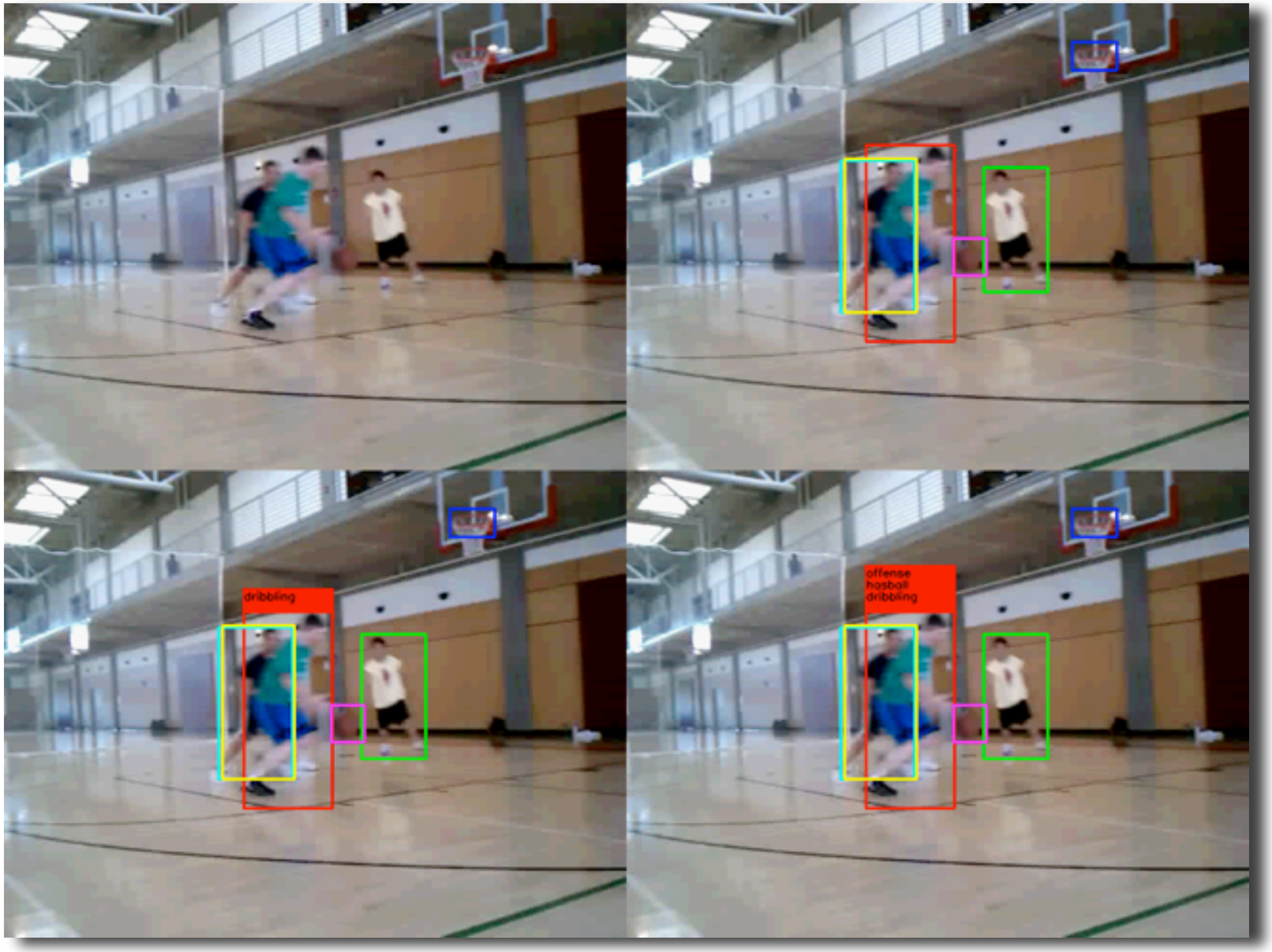
Probabilistic Logic CNF for Reasoning

Song-Chun Zhu, Sinisa Todorovic, and Ales Leonardis

At CVPR, Providence, Rhode Island
June 16, 2012

Goal



input

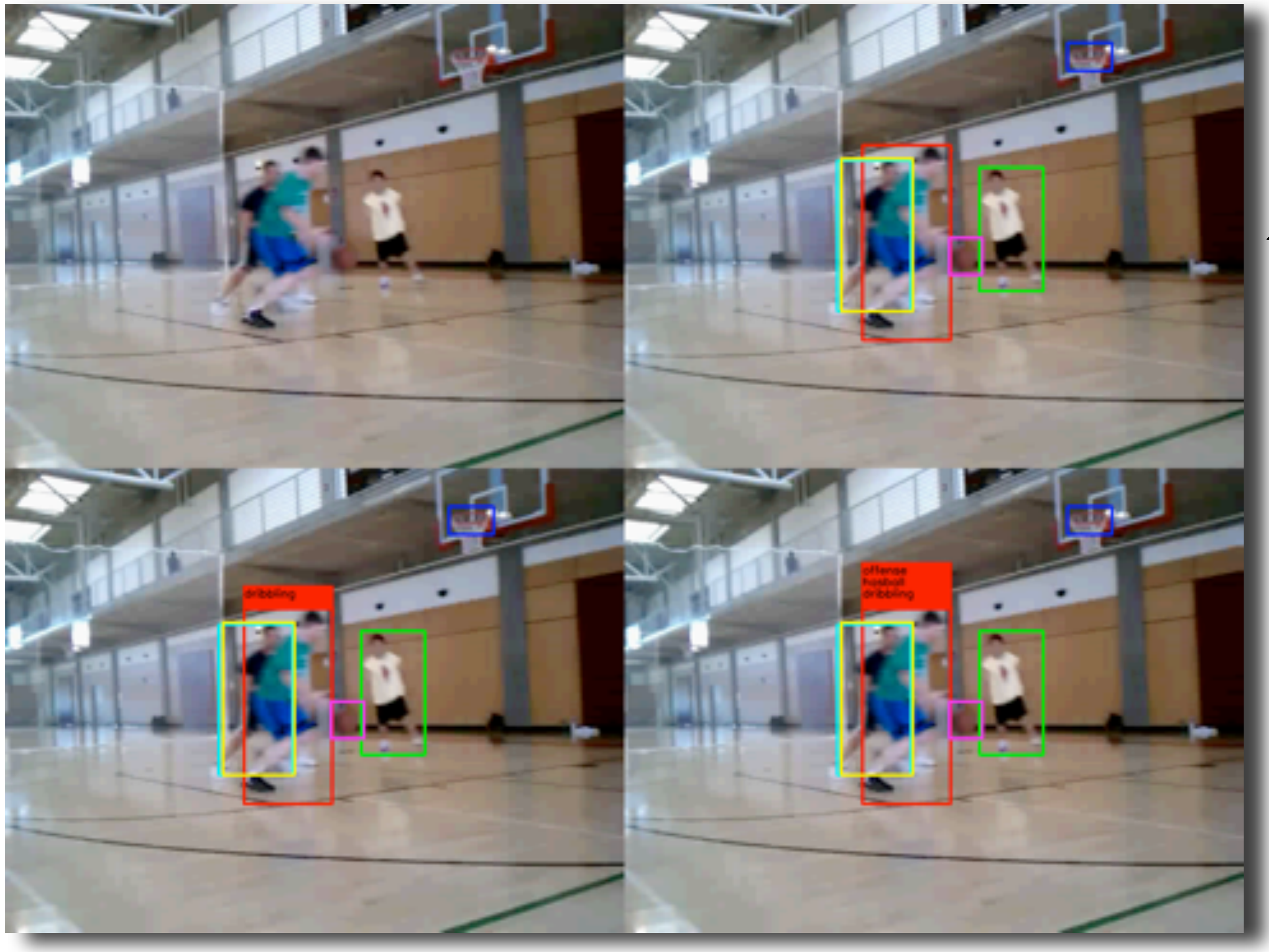
tracking

parsing

reasoning

activity recognition
in structured domains

Goal



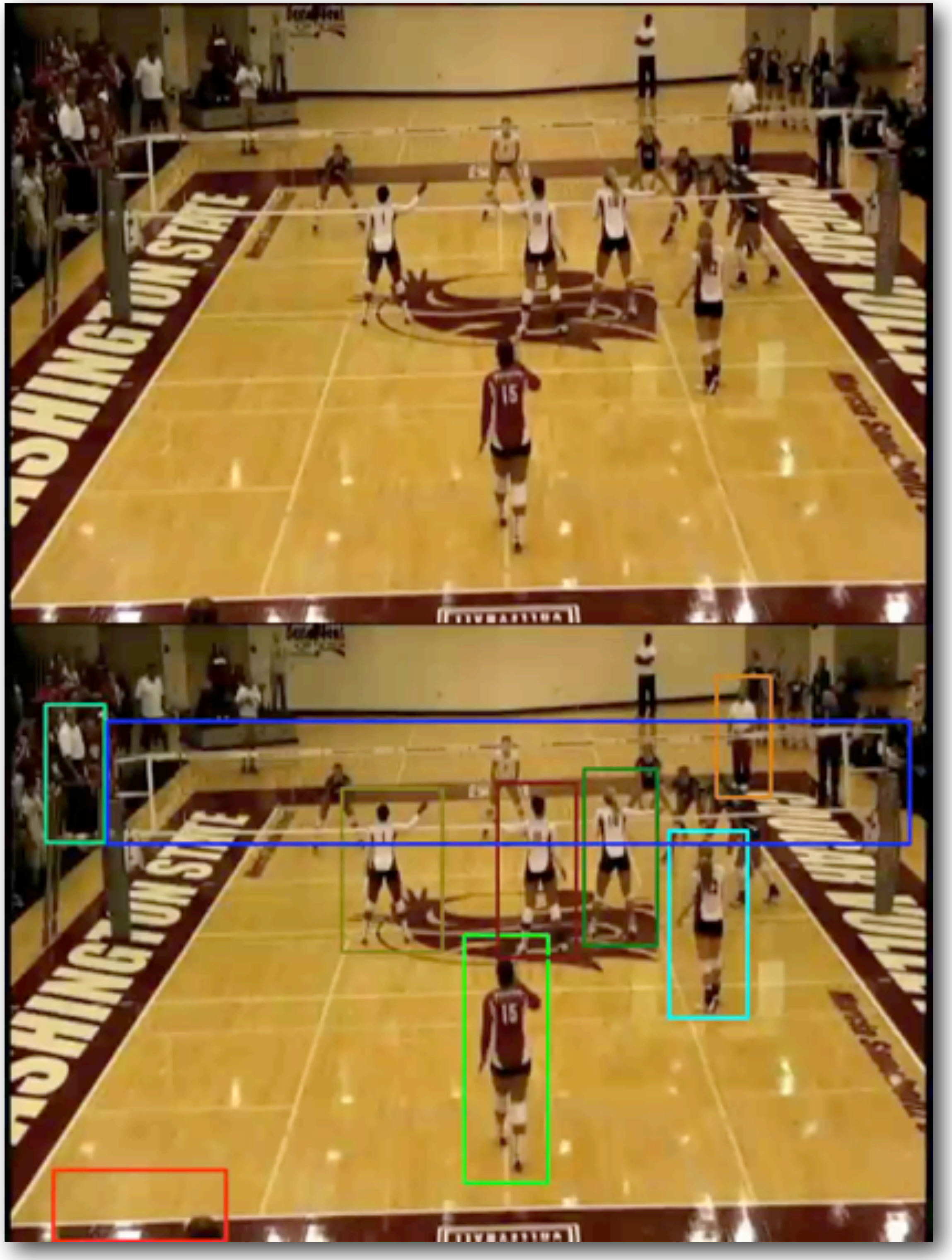
what: objects, events?

where, and **when**?

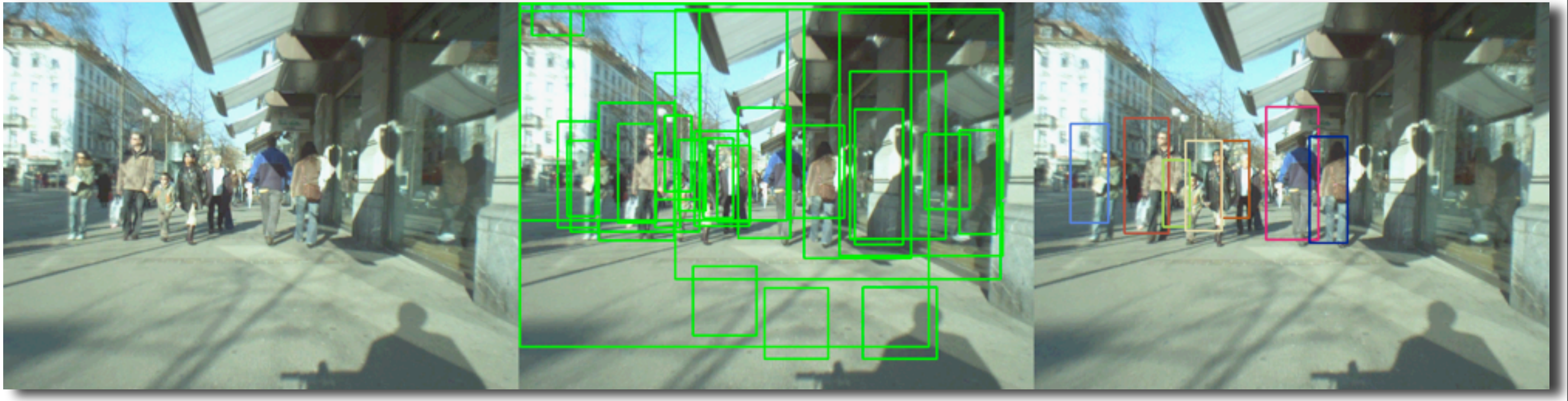
why: by explaining space-time relationships?

Three Semantic Levels of Events

- Primitive actions:
  - single actor-object interaction
  - punctual actions
  - repetitive actions
- Activity:
  - Short-term human-human-object interactions
  - e.g., passing the ball, hugging
- Events:
  - Long-term interactions of a group of people and objects



In this Talk: Tracking & Parsing are Given



“Multiobject Tracking as Maximum Weight Independent Set”

CVPR 2011

In Addition to Usual Challenges...

input

tracking

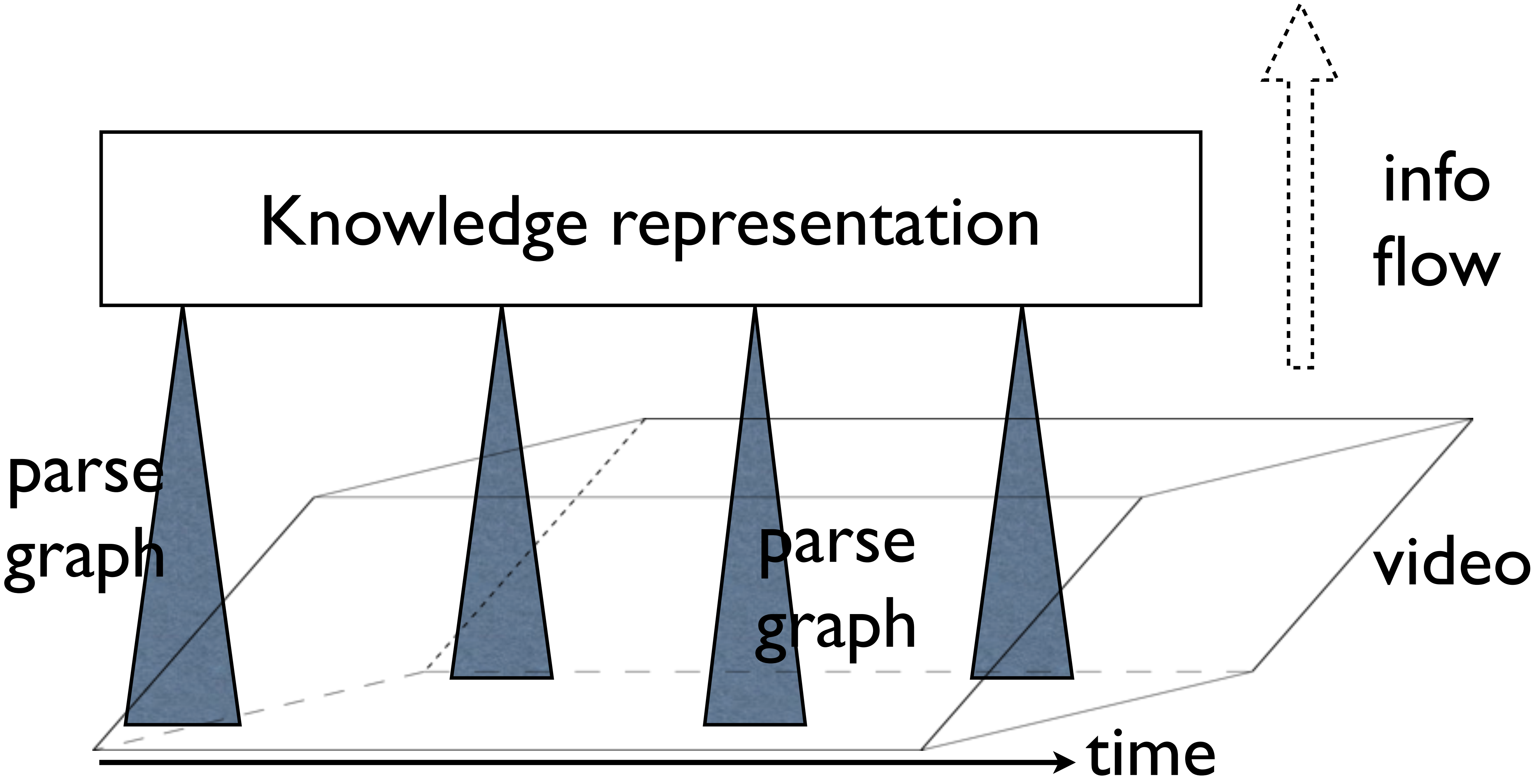
parsing

reasoning

- Who has the ball? -- partial occlusion
- Is the red player on offense? -- no direct cues
- Who violated the rules of basketball? -- domain

Two Key Ideas

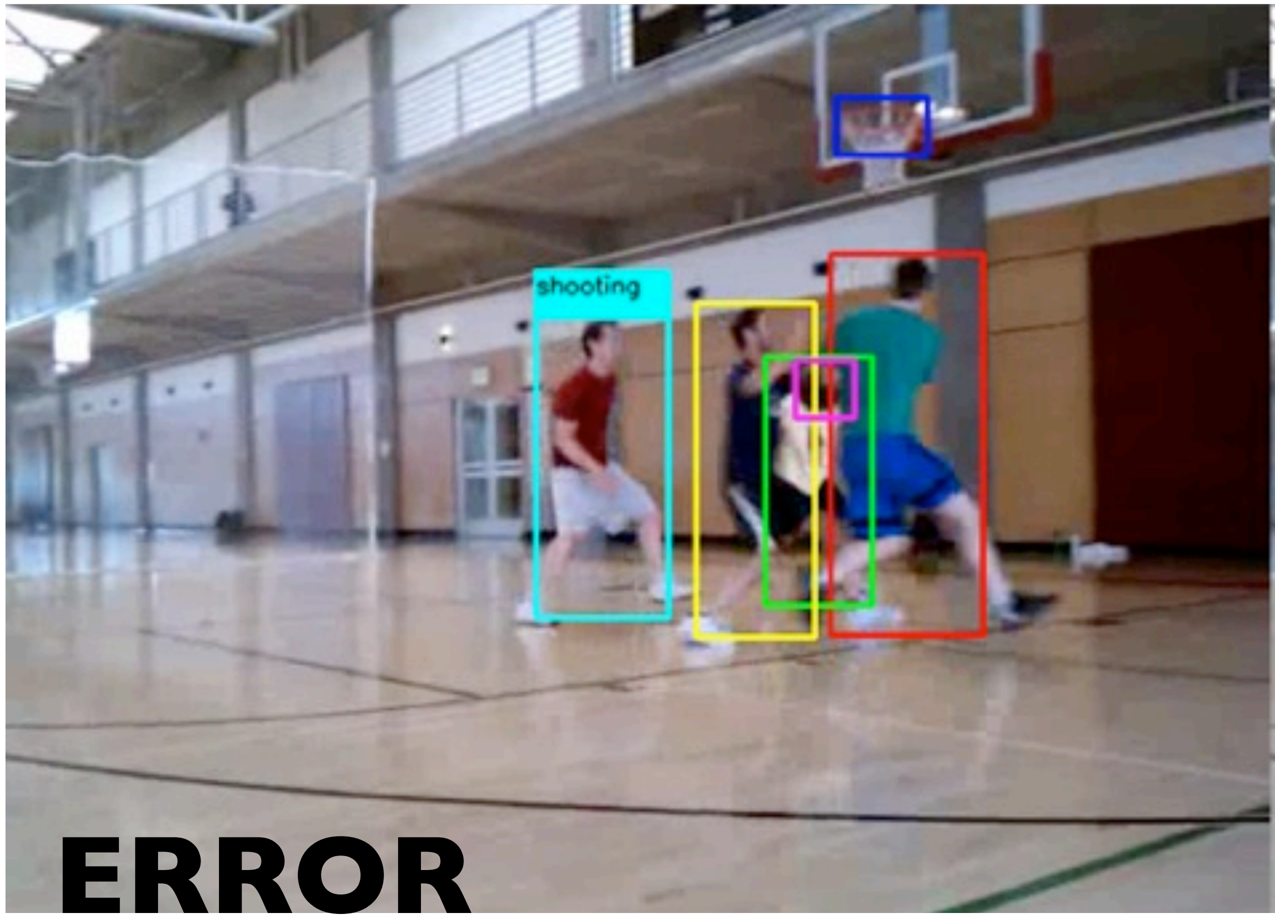
1. Ground reasoning onto parse graphs of primitive actions



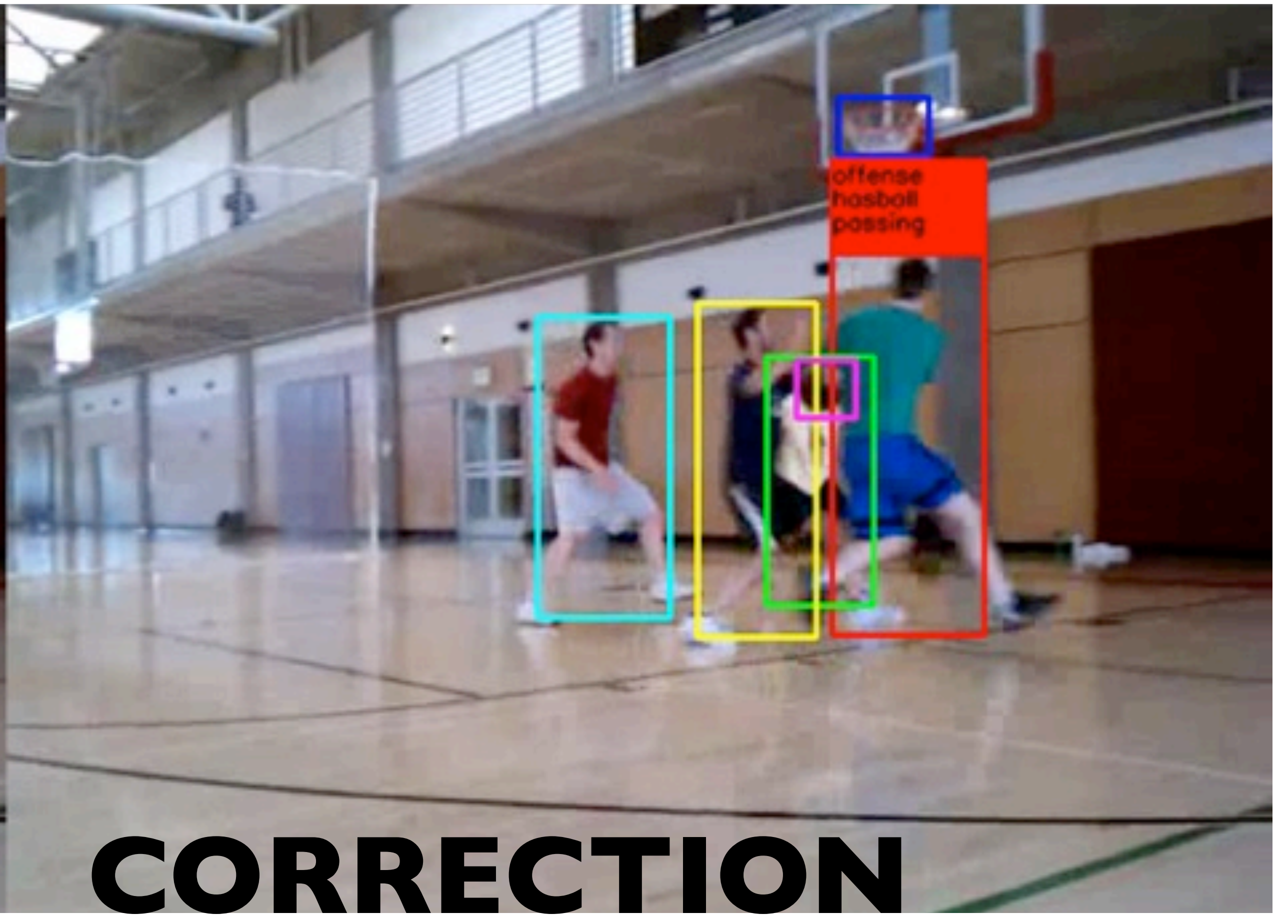
Two Key Ideas

2. Use domain knowledge to resolve uncertainty

   Top-down correction of errors in tracking, parsing, entity resolution



ERROR



CORRECTION

Knowledge Representation

- Probabilistic First-Order Logic
  - Rota & Thonnat '00 -- Declarative models
  - Siskind '01 -- Event logic
  - Nevatia et al. '04 -- Probabilistic ontology
  - Shet et al. '06 -- Multivalued logic
  - Richardson & Domingos '06 -- MLN
  - Shet et al. '07 -- Bilattice logic
  - Ryoo & Aggarwal '09 -- Space-time logic
  - Fern '09 - Penalty logic
  - Kersting & Raedt '11 -- Bayesian logic

Knowledge Base

$$\Sigma \ = \ \{(\phi_1, w_1), \ldots, (\phi_n, w_n)\}$$

a set of weighted logic formulas

$$w_n = P(\phi_n @ I)$$

a distribution of costs
of violating ϕ_n over
a time interval

Logic Formula

$$\mathrm{PassTo}(p, q) \rightarrow (\mathrm{Pass}(p) \wedge_m \mathrm{BallMoving} \wedge_m \mathrm{Catch}(q))$$

Event symbol:
e.g., interaction among a
number of object types

Syntax

$$\text{PassTo}(p, q) \rightarrow (\text{Pass}(p) \wedge_m \text{BallMoving} \wedge_m \text{Catch}(q))$$

Event symbol:
e.g., interaction among a
number of object types

Short-hand notation for:

$$\neg\phi \vee \phi'$$

Object types:
e.g., person, tool

Syntax

$$\mathrm{PassTo}(p, q) \rightarrow (\mathrm{Pass}(p) \wedge_m \mathrm{BallMoving} \wedge_m \mathrm{Catch}(q))$$

Event symbol:
e.g., interaction among a
number of object types

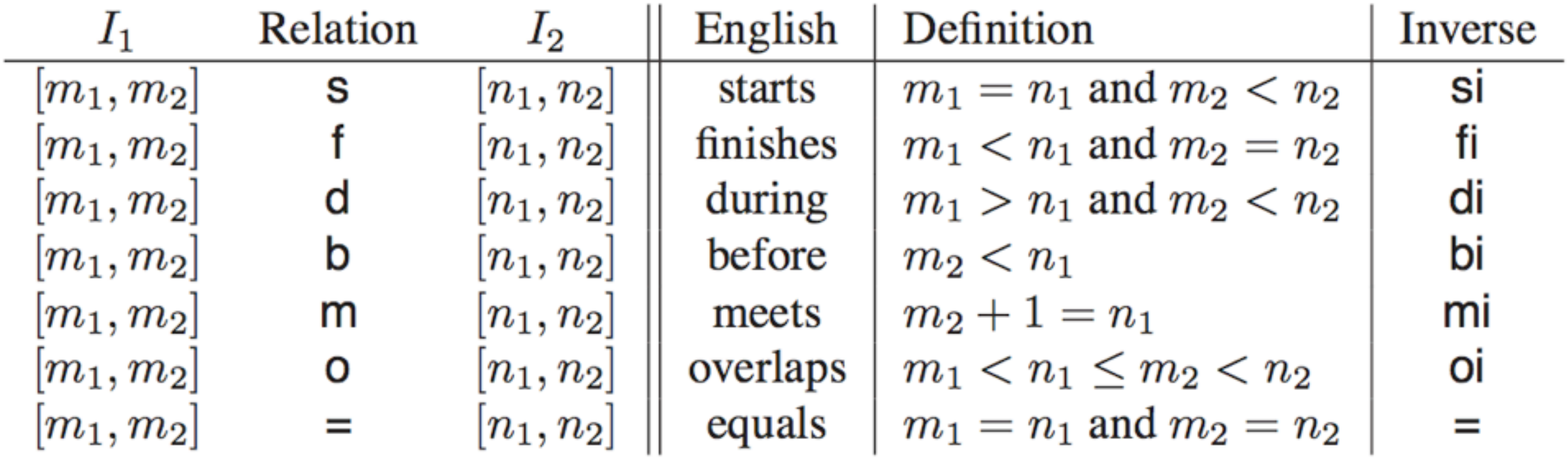
Three types of relations:

negation $\neg\phi$

disjunction $\phi \vee \phi'$

Temporal relations
between time intervals
where events are true $\phi \wedge_R \phi' \quad R \subseteq \mathbb{R}$

Allen Temporal Relations



I_1	Relation	I_2	English	Definition	Inverse
$[m_1, m_2]$	s	$[n_1, n_2]$	starts	$m_1 = n_1$ and $m_2 < n_2$	si
$[m_1, m_2]$	f	$[n_1, n_2]$	finishes	$m_1 < n_1$ and $m_2 = n_2$	fi
$[m_1, m_2]$	d	$[n_1, n_2]$	during	$m_1 > n_1$ and $m_2 < n_2$	di
$[m_1, m_2]$	b	$[n_1, n_2]$	before	$m_2 < n_1$	bi
$[m_1, m_2]$	m	$[n_1, n_2]$	meets	$m_2 + 1 = n_1$	mi
$[m_1, m_2]$	o	$[n_1, n_2]$	overlaps	$m_1 < n_1 \leq m_2 < n_2$	oi
$[m_1, m_2]$	=	$[n_1, n_2]$	equals	$m_1 = n_1$ and $m_2 = n_2$	=

time interval

Truth Values Assigned to Event Occurrences

$$\phi @ I$$

logic formula

time interval

observable
event occurrences

X: $\text{D-Dribbling}(P3)@[10, 30]$

from parse graphs

hidden
event occurrences

Y: $\text{Dribbling}(P3)@[20, 30]$

from reasoning

Interpretation

$$(X, Y) \models (\mathrm{HasBall}(P_1) \vee \mathrm{HasBall}(P_2))\ @[10, 20]$$

an event occurrence is true

along interval [10,20]

in interpretation (X, Y)

Model

Given KB: $\Sigma = \{(\phi_1, w_1), \ldots, (\phi_n, w_n)\}$

Prior distribution

confidence of the parse graph

$$P(X,Y|\Sigma) = \frac{1}{Z} \exp \left[\sum_{\phi_n \in \Sigma} \sum_{I_i} P(\phi_n @ I_i) P_{pg}(\phi_n @ I_i) P((X,Y) \models \phi_n @ I_i) \right]$$

likelihood that ϕ_n is true
in interpretation *(X,Y)*

all time intervals where ϕ_n is true

Reasoning = Most Probable Explanation

$$(X^*, Y^*) = \mathrm{MPE}(X, \Sigma) = \arg \max_{(X,Y)} P(X, Y | \Sigma)$$

We address intractable inference by:

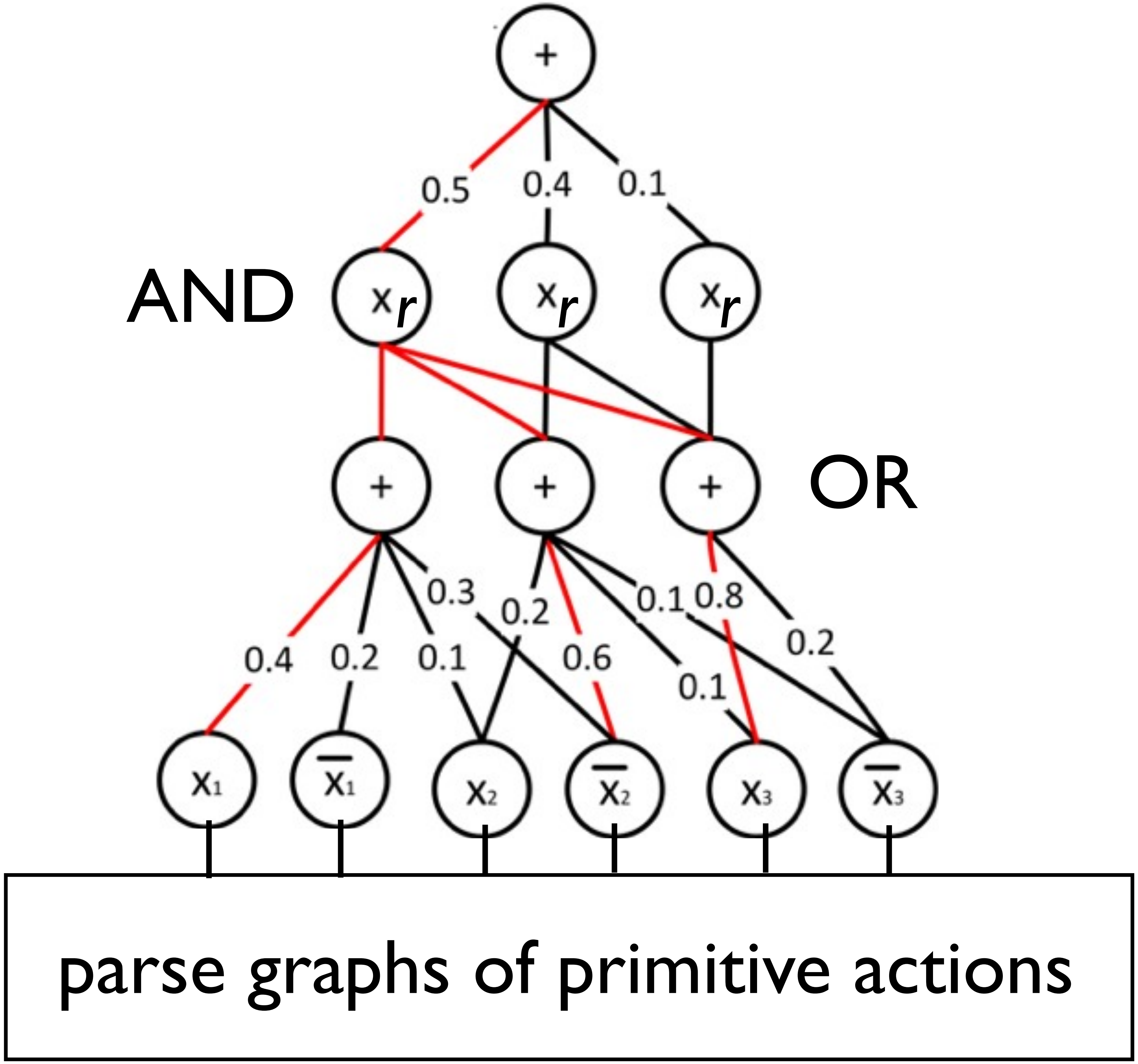
- Compiling Σ into CNF form => And-Or graph (AOG)
- Ensuring completeness and consistency of AOG
- Metropolis-Hastings moves over:
  - Logic formulas in Σ
  - Arguments of the logic formulas
  - Time intervals along which the formulas are true

Compilation of KB to AOG

Key idea:

Arithmetic circuit -- Data structure for efficient inference
Darwiche [2003]

Compilation of KB to AOG

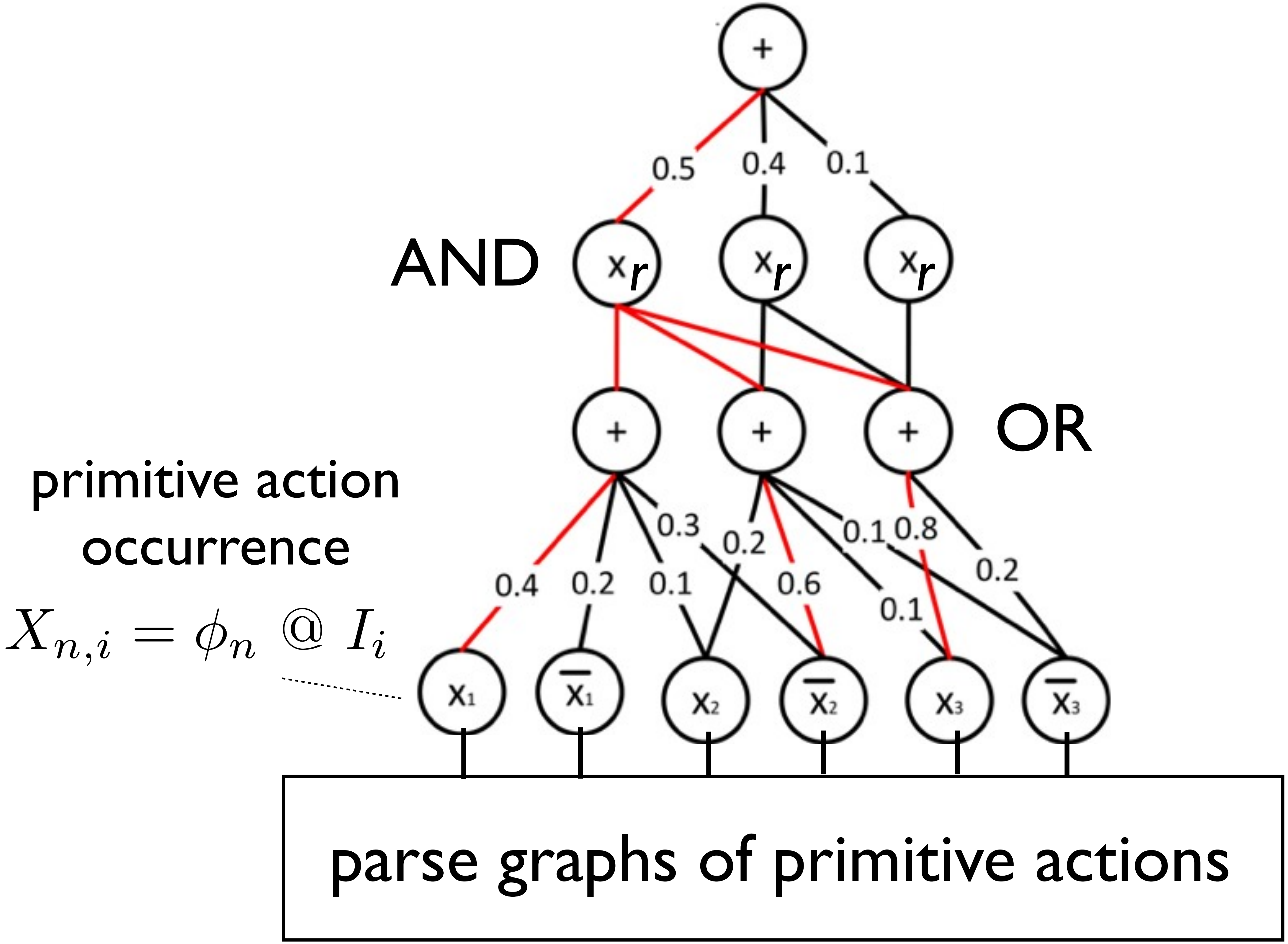


AND

OR

parse graphs of primitive actions

Compilation of KB to AOG



AND

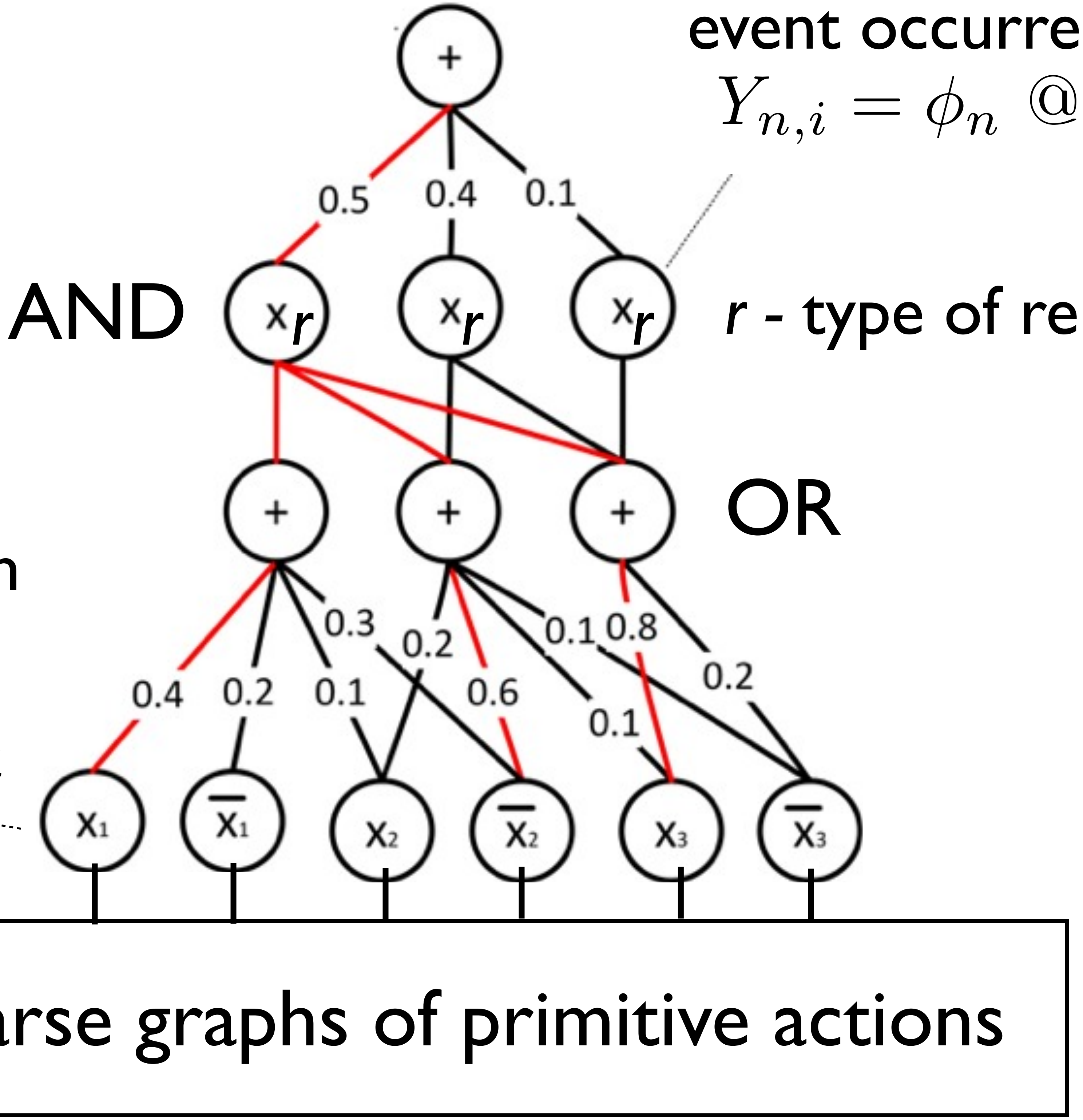
OR

primitive action occurrence

$$X_{n,i} = \phi_n \ @ \ I_i$$

parse graphs of primitive actions

Compilation of KB to AOG



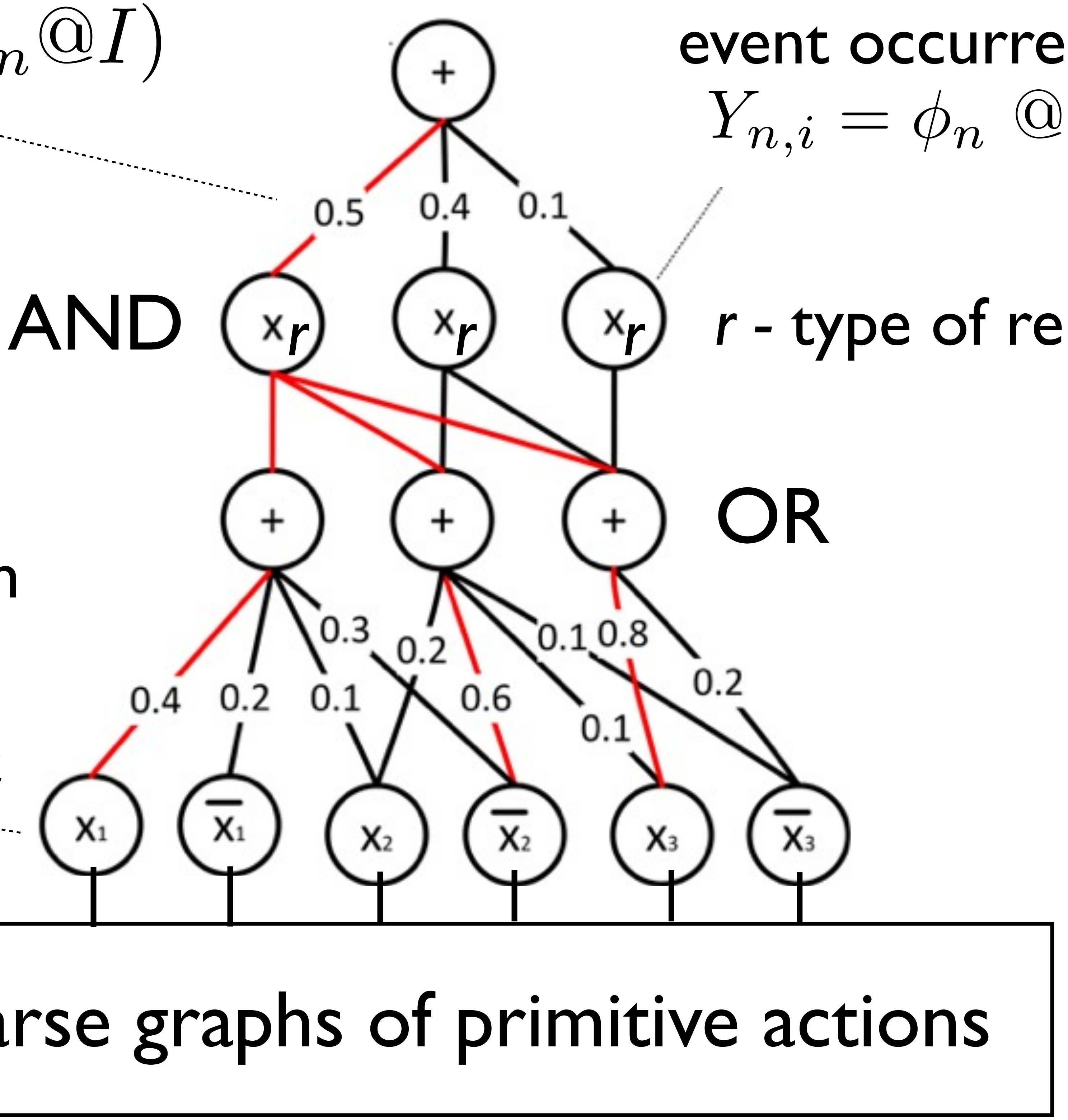
event occurrence
$Y_{n,i} = \phi_n \ @\ I_i$

r - type of relationship

primitive action occurrence
$X_{n,i} = \phi_n \ @\ I_i$

parse graphs of primitive actions

Compilation of KB to AOG



$w_n = P(\phi_n @ I)$

event occurrence

$Y_{n,i} = \phi_n \ @ \ I_i$

AND

r - type of relationship

OR

primitive action
occurrence

$X_{n,i} = \phi_n \ @ \ I_i$

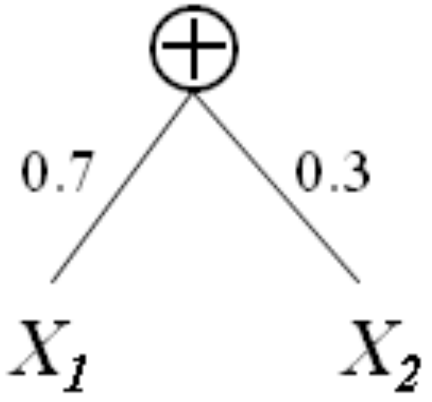
parse graphs of primitive actions

Valid Compilation

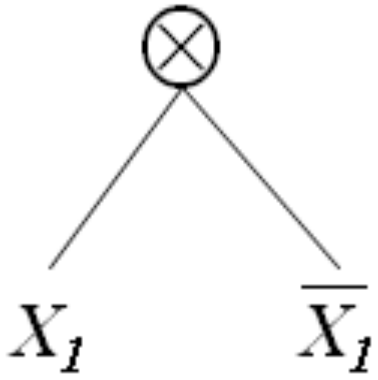
Theorem: AOG is valid iff it is complete and consistent

Complete: Under sum, children cover the same set of variables

Consistent: Under product, no variable in one child and negation in another



Incomplete



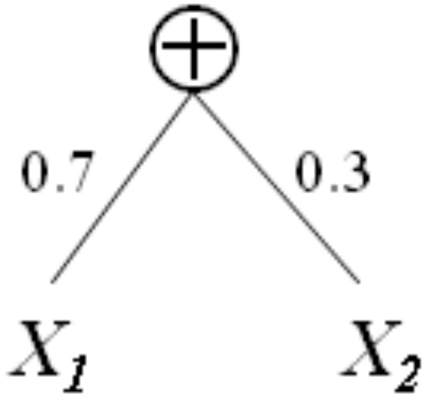
Inconsistent

Efficiency

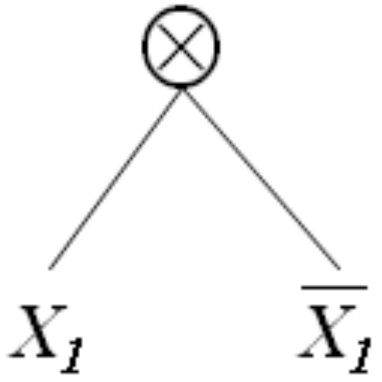
Theorem: Valid AOG allows polynomial inference in the number of nodes

Complete: Under sum, children cover the same set of variables

Consistent: Under product, no variable in one child and negation in another



Incomplete

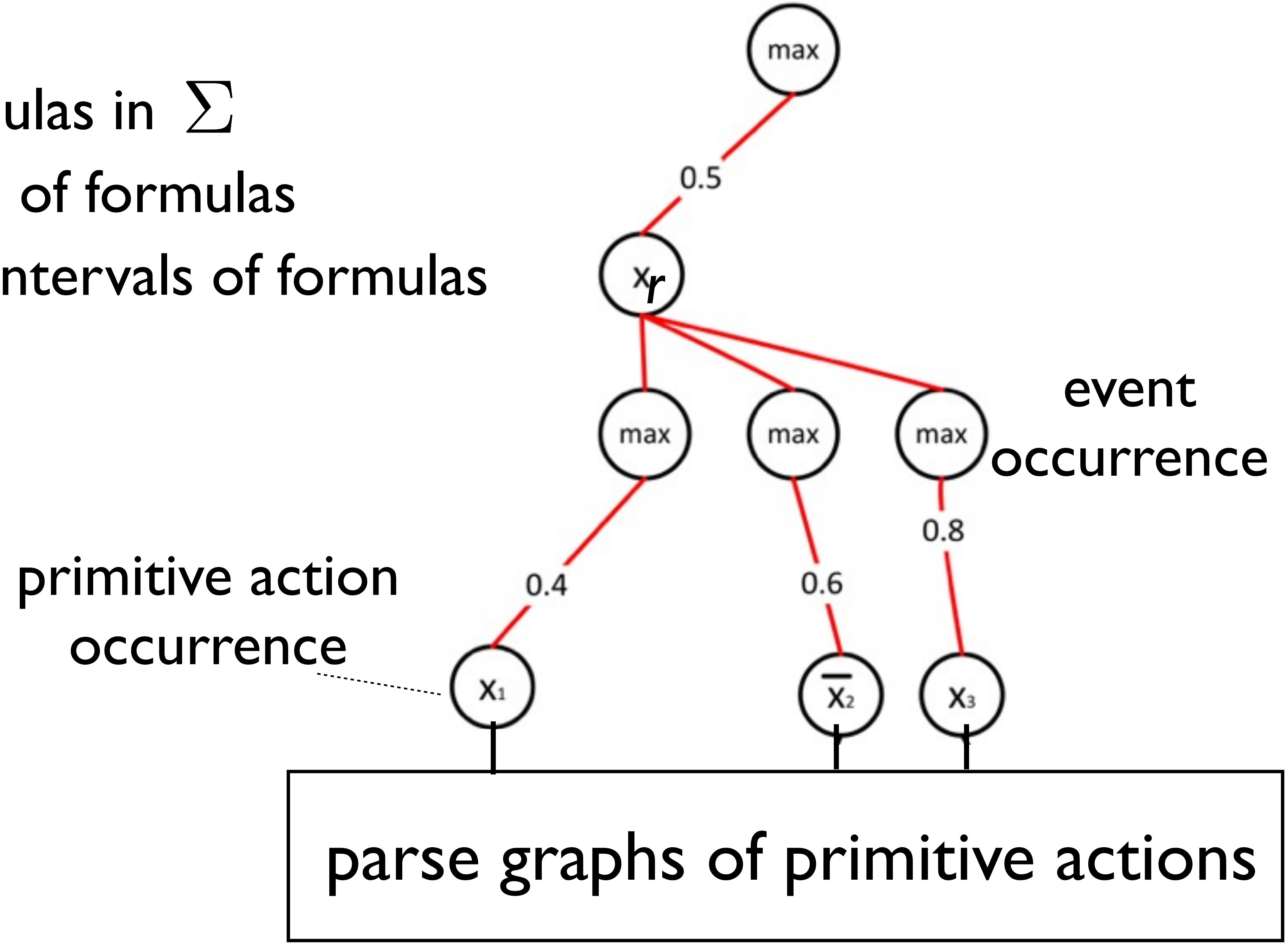


Inconsistent

Most Probable Explanation

Identifies:

- Logic formulas in Σ
- Arguments of formulas
- True time intervals of formulas



event occurrence

primitive action occurrence

parse graphs of primitive actions

Metropolis-Hastings Moves

two probable interpretations $A = (X, Y)_A \qquad B = (X, Y)_B$

$$\alpha(A \rightarrow B) = \min\left(1, \frac{Q(B \rightarrow A)P(B|G)}{Q(A \rightarrow B)P(A|G)}\right)$$

proposal distribution

efficient proposals of time intervals
without enumerating exponentially many
subintervals of all intervals

compiled KB
into AOG

CVPR 2011

Scheduling the Moves -- Open Problem

How to prioritize particular moves over:

- Logic formulas in Σ
- Arguments of formulas
- True time intervals of formulas

Our approach:

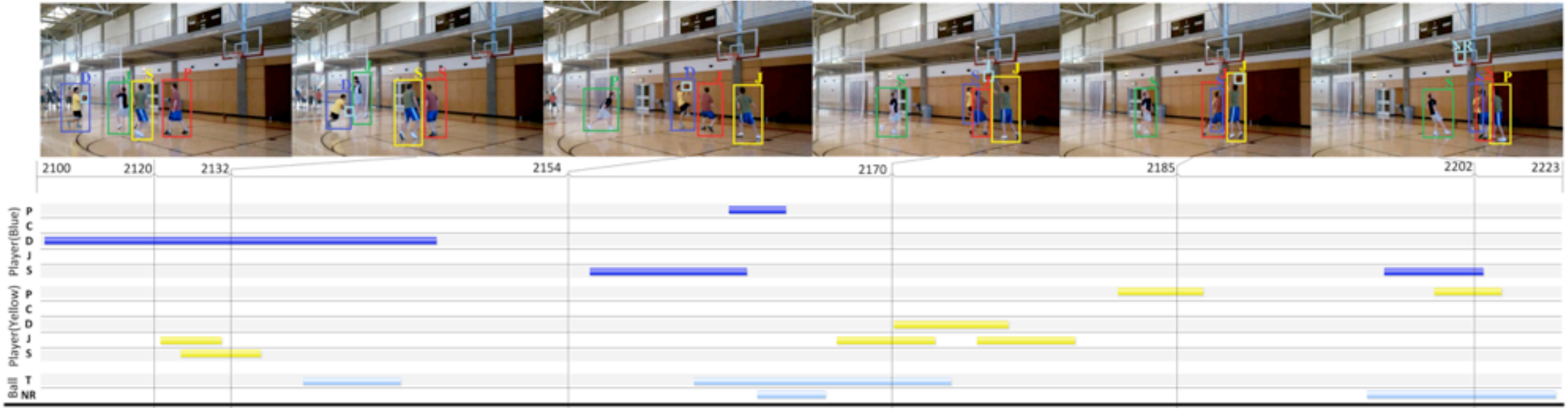
1. Map the current interpretation into a feature vector

$$(X, Y)_A \rightarrow \Psi_A$$

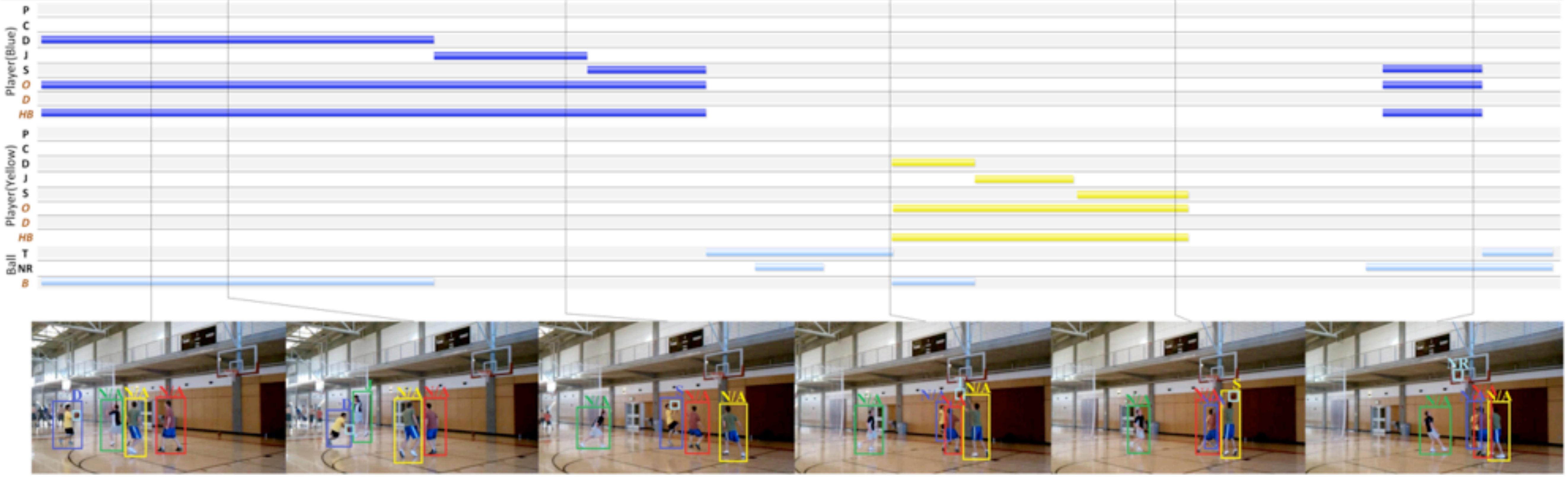
2. Classify the feature vector

Results -- CVPR 11

input parsing results

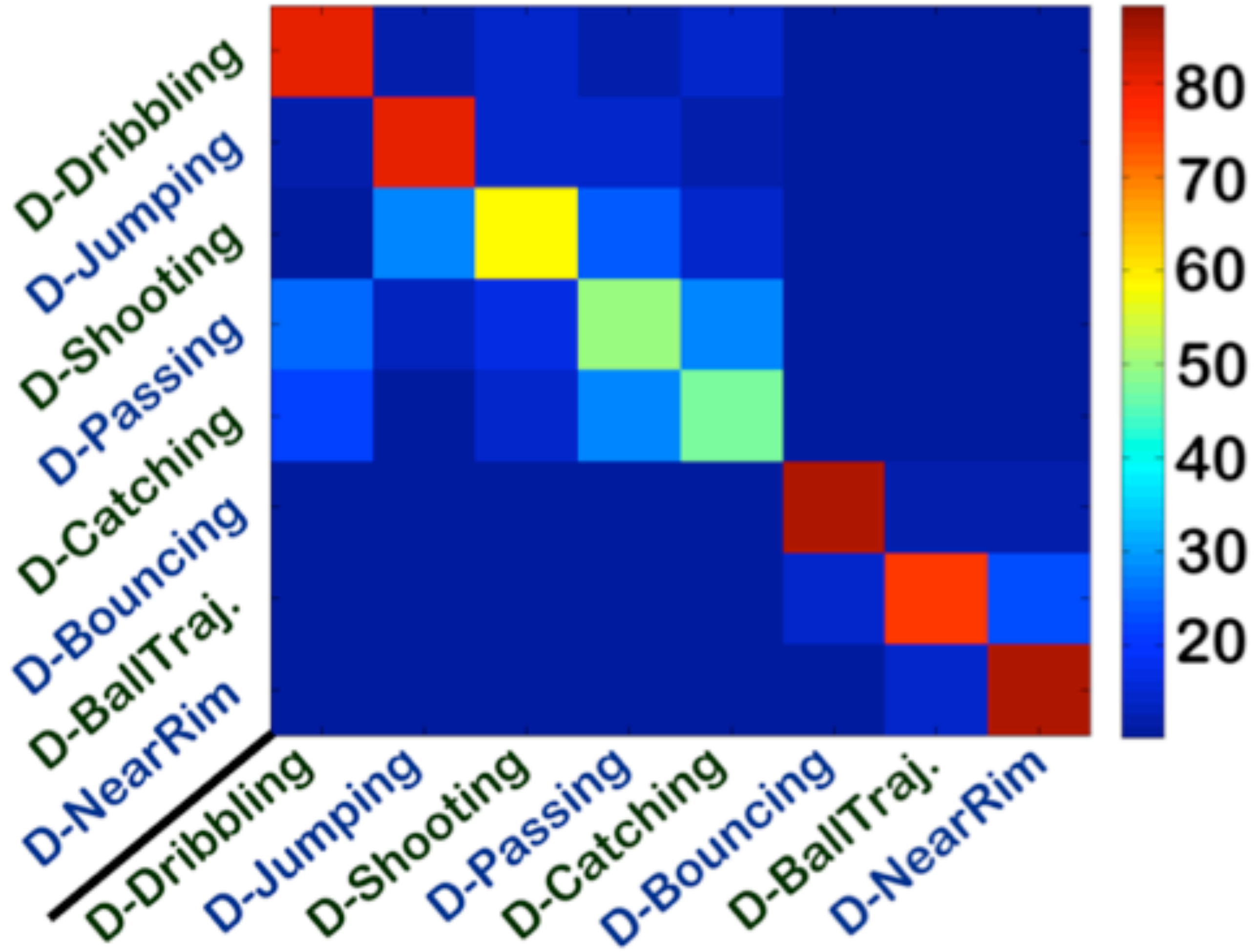


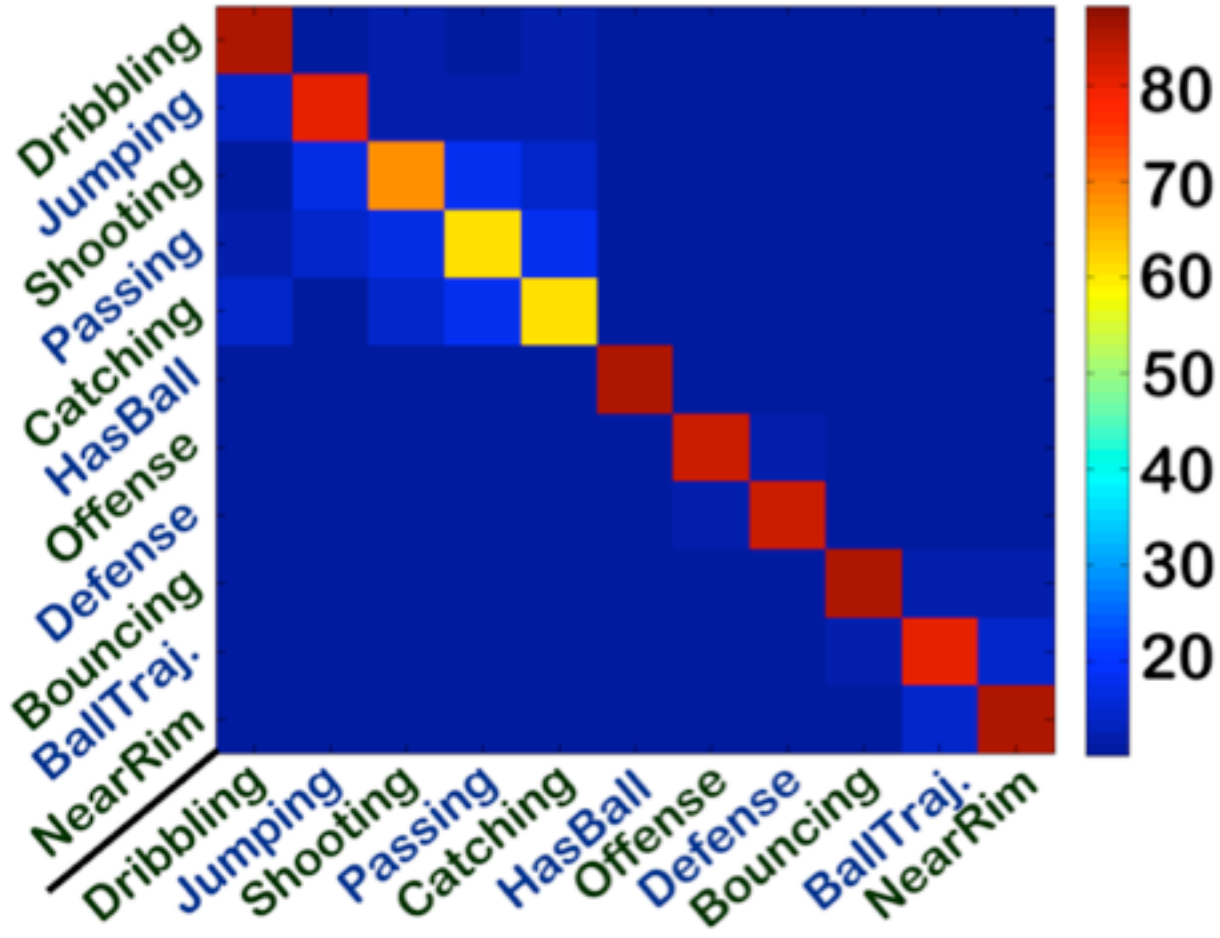
reasoning: most probable explanation



Results -- CVPR 11

Confusion tables

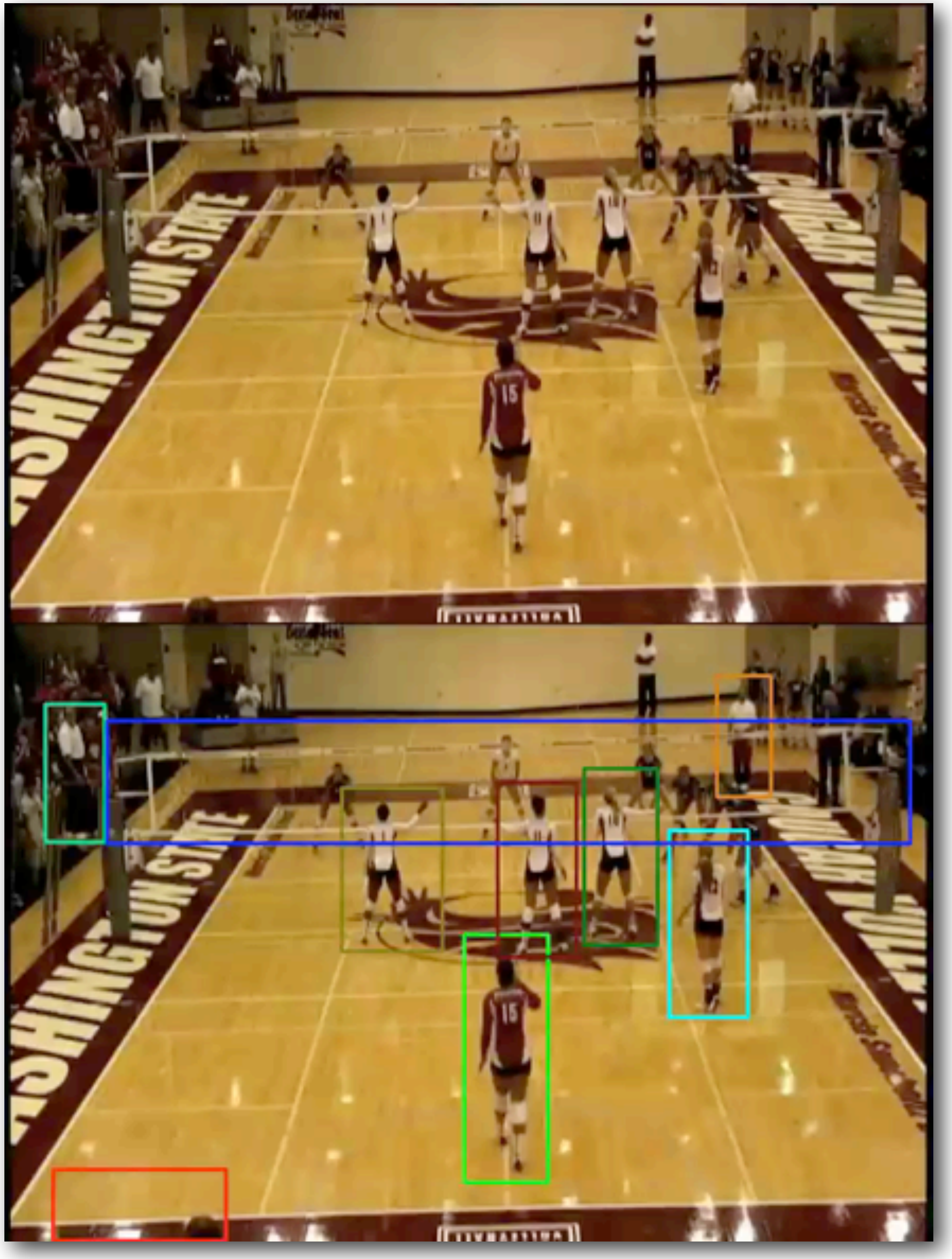




input parsing results
of primitive actions

reasoning including
higher-level events

Summary



Reasoning helps:

- correct tracking/parsing errors
- disambiguate uncertainty
- address higher-level events

THANK YOU